



**West Byfleet Junior School**

**Written Calculations**

**2023-2024**

## **CONTENTS**

Introduction

Whole School Approach

Progressions in Addition

Progressions in Subtraction

Progressions in Multiplication

Progressions in Division

Summary

## INTRODUCTION

The National Curriculum provides a structured and systematic approach to teaching number with a considerable emphasis placed on teaching written calculation strategies. Once the child feels confident with their chosen mental method, they will begin to use formal written methods in Year 3.

## REASONS FOR USING WRITTEN METHODS:

- To aid mental calculation by writing down some of the numbers and answers involved
- To make clear a mental procedure for the pupil
- To help communicate methods and solutions
- To provide a record of work to be done
- To aid calculation when the problem is too difficult to be done mentally
- To develop and refine a set of rules for calculation

## WHOLE SCHOOL APPROACH

We have developed a consistent approach to the teaching of written calculation methods. This will establish continuity and progression throughout the school.

**Mental methods will be established. These will be based on a solid understanding of place value in number and include the following:**

- I. Remembering number facts and recalling them without hesitation, *e.g. pairs of numbers which make 10, doubles and halves to 20*
- II. Using known facts to calculate unknown facts, *e.g.  $6+6=12$  therefore  $6+7=13$*
- III. Understanding and using relationships between addition and subtraction to find answers and check results, *e.g.  $14+6=20$  therefore  $20-6=14$*

- IV. Having a repertoire of mental strategies to solve calculations,  
*e.g. doubles/ near doubles,*  
*bridging 10/ bridging 20,*  
*adding 9 by +10 & -1*
- V. Making use of informal jottings such as blank number lines to assist in calculations with larger numbers, *e.g.  $83-15=68$*
- VI. Solving one-step word problems (either mentally or with jottings) by identifying which operation to use, drawing upon their knowledge of number bonds and explaining their reasoning
- VII. Beginning to present calculations in a horizontal format and explain mental steps using numbers, symbols or words
- VIII. Learn to estimate/ approximate first, *e.g.  $29+30$  (round up to the nearest 10, the answer will be near to 60)*

The empty number line will be used in all years, but particularly in Year 3, to aid calculations.

Subtraction will be taught by counting back or counting on depending on the numbers.

At every stage children will be encouraged to estimate the answer first.

## **WHEN ARE CHILDREN READY FOR WRITTEN CALCULATIONS?**

The New National Curriculum states that children should use a formal written method, however, we must ensure the children are confident with using a mental strategy for addition, subtraction, multiplication and division.

### **Addition and Subtraction**

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two-digit numbers mentally?
- Can they explain their mental strategies orally and record them using formal jottings?

### **Multiplication and Division**

- Do they know the 2, 3, 4, 5 and 10 times tables?
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a placeholder?

- Do they understand the commutative law? (e.g.  $8 \times 7 = 7 \times 8$ )
- Can they multiply two- digit and three-digit numbers by 10 and 100?
- Can they double and halve two-digit numbers mentally?
- Can they use known multiplication facts to derive mentally other multiplication facts they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

All teachers will use their own teacher judgement to decide which methods are most beneficial for their children. Although the contracted methods are generally used towards upper Key Stage 2, a teacher may decide this can be introduced earlier or later, depending on the abilities of the children they are teaching. Methods taught will be chosen to support the class in the most appropriate manner.

Click on any of the method names in the sections below to take you to the school website. Here you will be able to see a video of how the method works.

## PROGRESSIONS IN ADDITION

All the below addition methods can be used to add:

+

$$\begin{array}{r} 400 \quad 60 \quad 6 \\ 300 \quad 50 \quad 8 \\ \hline \end{array}$$

### 1. Expanded column addition

$$\begin{array}{r} 400 \quad 60 \quad 6 \\ + 300 \quad 50 \quad 8 \\ \hline 700 + 110 + 14 = 824 \end{array}$$

$$466 + 358 =$$

Partition each digit into each place value e.g. hundreds, tens and ones.

Then add each place value columns together.

Finally add your answers together.

$$466 + 358 =$$

Partition the numbers into each place value e.g. hundreds, tens and ones.

Then add each place value column together- this time if your answer exceeds the value of the column e.g. 10, 100, 1000 then you need to carry across the additional amount.

E.g.  $6 + 8 = 14$  (More than 10) which is the same as  $10 + 4$ .

The 4 stays under the ones column and the 10 gets carried under the tens column (below the answer box) - Don't forget to cross the carried amount out after adding it to the column's total.

Finally add each column's answers together.

$$\begin{array}{r} 400 \quad 60 \quad 6 \\ + 300 \quad 50 \quad 8 \\ \hline 800 \quad + \quad 20 \quad + \quad 4 \quad = 824 \\ \cancel{100} \quad \cancel{10} \end{array}$$

The above methods can be used for any number of digits.

## 2. Contracted column addition

$$\begin{array}{r}
 47 \\
 + 76 \\
 \hline
 13 \\
 110 \\
 \hline
 123
 \end{array}$$

$$47 + 76 =$$

First of all you add the ones digits together e.g.  $6 + 7 = 13$  and place the answer under the correct place value.

Next you add the tens digits together e.g.  $40 + 70 = 110$  and place below your first calculation.

Finally you add the amounts together e.g. 13 and 110

$$\begin{array}{r}
 368 \\
 + 494 \\
 \hline
 12 \\
 150 \\
 \hline
 700 \\
 862
 \end{array}$$

*This can be used for any number of digits too.*

$$\begin{array}{r}
 47 \\
 + 76 \\
 \hline
 123 \\
 \hline
 \cancel{1} \quad \cancel{1}
 \end{array}$$

This time the additional amount gets carried below the column to the left. E.g.

$$7 + 6 = 13.$$

13 is not possible underneath one place value heading as you cannot have 13 ones. You have to exchange ten of the ones for one ten and have 3 ones remaining.

The 3 stays under the ones column but 1 (meaning one ten) goes under the tens column (below the answer box) to be added on to the tens digit calculation.

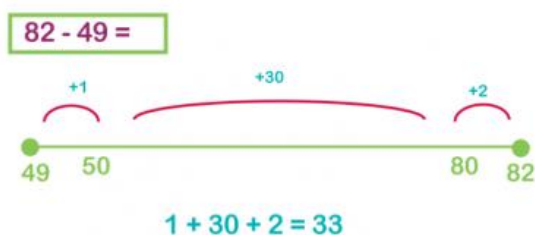
Don't forget to cross through the carried amount once you have added it.

$$\begin{array}{r}
 368 \\
 + 493 \\
 \hline
 861 \\
 \hline
 \cancel{1} \quad \cancel{1}
 \end{array}$$

## PROGRESSIONS IN SUBTRACTION

All the below subtraction methods can also be used when subtracting decimals and money.

### 1. Subtracting on a number line by counting on.



$$82 - 49.$$

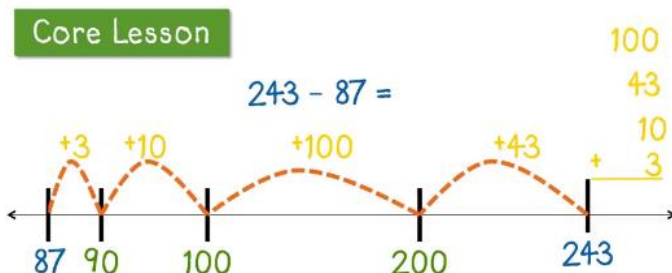
First draw a number line and put your smaller number at the start and your larger number at the end.

Next make a jump from the smaller number to the next ten e.g.  $49 \rightarrow 50$ . Record how far you have jumped either above or in your 'jump'

Next, count in tens or groups of 10 to the nearest ten before your final number e.g. jump to 80.

Then jump from your multiple of 10 to your final number e.g.  $80 \rightarrow 82$

Finally add all your jumps together.



### 2. Expanded column subtraction

$$\begin{array}{r} 500 \\ - 200 \\ \hline 300 \end{array} \quad \begin{array}{r} 60 \\ - 40 \\ \hline 20 \end{array} \quad \begin{array}{r} 3 \\ - 1 \\ \hline 2 \end{array}$$

$$563 - 241 =$$

Partition each number into each place value e.g. hundreds, tens and ones.

Then subtract each place value column

$$932 - 457 =$$

Partition each number into each place value e.g. hundreds, tens and ones.

Then subtract each place value column.

If the number above is smaller than the number below it e.g. 2 is less than 7, then you need to exchange/ borrow from the column to the left.

You must exchange 1 ten for 10 ones from the tens column and add it to the ones column. You now only have 20 in the tens column and you now have 12 ones.  $12 - 7 = 5$

Now you have 20 tens - 50 tens, which you can't do. You have to borrow 1 hundred from the hundreds column and exchange for 10 tens. You now have 800 in the hundreds column and 120 in the tens column.  $120 - 50 = 70$

$$\text{Next } 800 - 400 = 400$$

Finally add each column's answers together.

$800 \quad 120$

$900 + 100$

$- 400 + 50 + 7$

$400 + 70 + 5$

### 3. Contracted column subtraction

$$\begin{array}{r} 5 \quad 6 \quad 3 \\ - 2 \quad 4 \quad 1 \\ \hline 3 \quad 2 \quad 2 \end{array}$$

This time you subtract in columns.

Always start with the ones columns

E.g.  $3 - 1 = 2$  then move on to the tens column

$6 - 4 = 2$  then the hundreds column  $5 - 2 = 3$

$$\begin{array}{r} \overset{8}{\cancel{9}} \overset{12}{\cancel{3}} \overset{1}{\cancel{2}} \\ - 457 \\ \hline 475 \end{array}$$

You subtract each place value column- starting with the ones.

If the number above is smaller than the number below it

e.g. 2 is less than 7, then you need to exchange/ borrow from the column to the left.

You must exchange 1 ten for 10 ones from the ten column and add it to the ones column. You now only have 2 tens and you now have 12 ones.  $12 - 7 = 5$

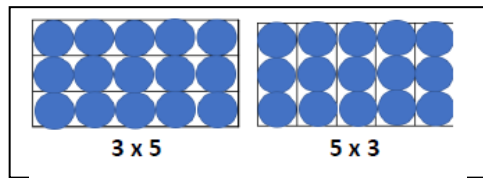
Now you have 2 tens – 5 tens, which you can't do. You have to exchange 1 hundred from the hundreds column and exchange for 10 tens. You now have 8 in the hundreds column and 12 in the tens column.  $12 - 5 = 7$

Next  $8 - 4 = 4$



## PROGRESSIONS IN MULTIPLICATION

### 1. Arrays



If your calculation is  $3 \times 5$  you would record it by creating 3 rows of 5 circles in each.

You could also do  $5 \times 3$  by creating 5 rows of 3 circles in each.

To work out the total you could either count in your 5 or 3 times table. Or count the individual circles/ crosses that you have used to illustrate your calculation

### 2. Grid Method

×	20	3	
4	80	12	= 92

When solving using the grid method you need to first partition your numbers to lay out your calculation.  
 $23 \times 4$ .

You then multiply the two numbers (that link to the box) together  
e.g.  $20 \times 4 = 80$  and  $3 \times 4 = 12$

You then add your totals together (using column addition if needed)  
 $80 + 12 = 92$

×	200	50	3	
6	1200	300	18	= 1518

For harder calculations such as  $6 \times 200$  you can use known facts to help you e.g.  $6 \times 2$ . You have made 200, one hundred times smaller therefore you need to multiply your answer by 100 at the end.

×	10	6	
40	400	240	= 640
8	80	48	= 128
			768

$6 \times 2 = 12$   
 $6 \times 200 = 12 \times 100 = 1200$

### 3. Expanded Column method

$$\begin{array}{r}
 38 \\
 \times 7 \\
 \hline
 \cancel{56} \quad (7 \times 8) \\
 + 210 \quad (7 \times 30) \\
 \hline
 266
 \end{array}$$

$$\begin{array}{r}
 56 \\
 \times 27 \\
 \hline
 42 \quad (7 \times 6) \\
 350 \quad (7 \times 50) \\
 120 \quad (20 \times 6) \\
 + \quad \quad \quad (20 \times 50) \\
 1000 \\
 \hline
 1512
 \end{array}$$

When solving using the expanded method- each individual calculation gets placed in brackets next to each total. Each digit of one number has to be multiplied by every digit of the other number.

All totals (laid out under the correct place value heading) then get added together for a complete total.

**First we multiply each of the digits 391 by 9.**

$9 \times 1 = 9$   
 $9 \times 9 = 81$  (put the 1 down; carry the 8)  
 $9 \times 3 = 27$   
 $27 + (\text{carried } 8) = 35$

**Now we multiply each of the digits 391 by 3. Because it is actually 30, not 3, we put a zero down first.**

$3 \times 1 = 3$   
 $3 \times 9 = 27$  (put the 7 down and carry the 2)  
 $3 \times 3 = 9$  (plus the 2 which makes 11)

**Last of all, we add the results of our calculations to get the answer.**

$3519 + 11730 = 15249$

*Extend to Decimals*

## PROGRESSIONS IN DIVISION

### 1. Division through sharing and grouping

Share 35 tallies between 5 groups...

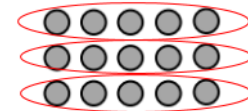


How many tallies in each group?

$$35 \div 5 =$$



#### Array Model Division and Multiplication

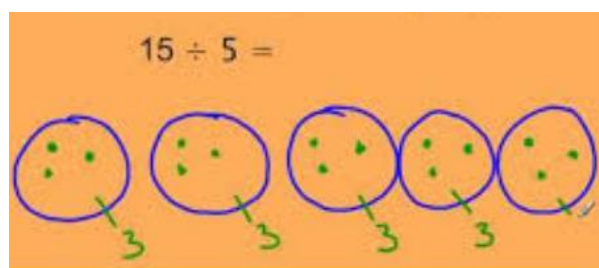


$$15 \div 3 = 5$$

Total number  
Number of groups  
Number in each group

$$3 \times 5 = 15$$

Number of groups  
Number in each group  
Total number



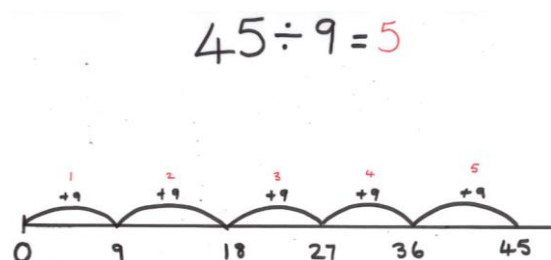
You can divide through sharing by placing tallies into groups.

Either sharing one tally into five separate groups and continue sharing tallies between them until the total number of tallies have been recorded (in this case 35).

Or if the question had been  $35 \div 5$  you could create groups of 5 tallies until you reached 35, resulting in 7 groups of 5 tallies.

This can be done by sharing into groups (circles) too as well as in arrays (making a link to multiplying)

### 2. Division on a number line



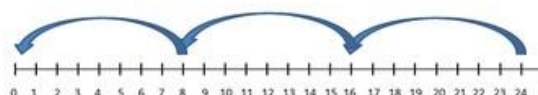
You can divide by using a number line.

You can either start at 0 and count on by the divisor (9) until you reach your dividend (45) to reach your answer (5) or you could start at your dividend and count back.

If you cannot count the whole way to the dividend or get all the way back to zero, then your answer has a remainder.

Let's show repeated subtraction on a number line.

$$24 \div 8 = 3$$



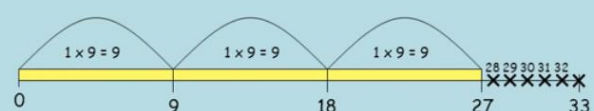
Start at 24.

Count back by 8 until you get to zero.

How many times did we subtract 8?



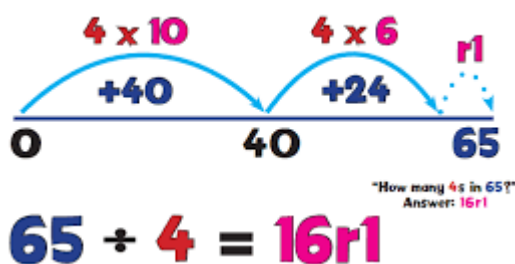
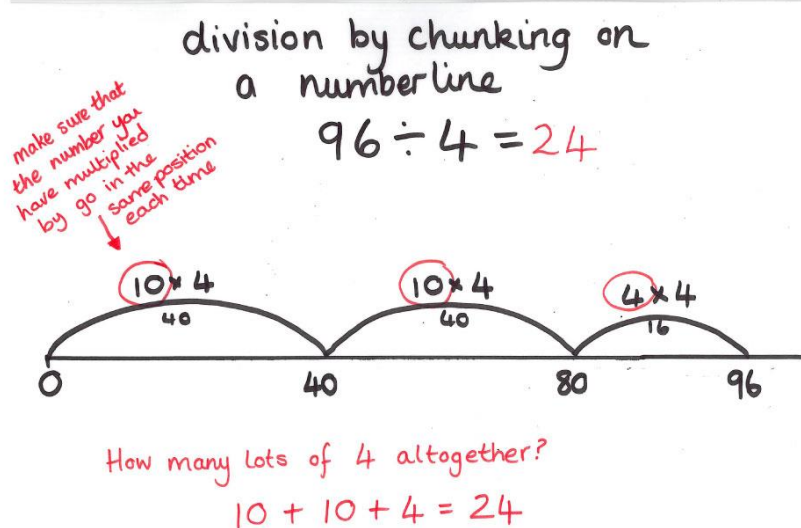
$33 \div 9 = ?$  How many lots of 9 are there in 33?



There are 3 lots of 9 in 33 with 6 remaining,

$$\text{So } 33 \div 9 = 3 \text{ remainder } 6 \\ = 3 \text{ r}6$$

### 3. Division by chunking on a number line



<b>Division</b>  An operation that finds how many things are divided equally into groups	<b>Divisor</b> $20 \div 5 = 4$ The number that does the dividing.
<b>Dividend</b> $20 \div 5 = 4$ The number that will be divided.	<b>Quotient</b> $20 \div 5 = 4$ The answer to a division equation.

Division by chunking on a number line.

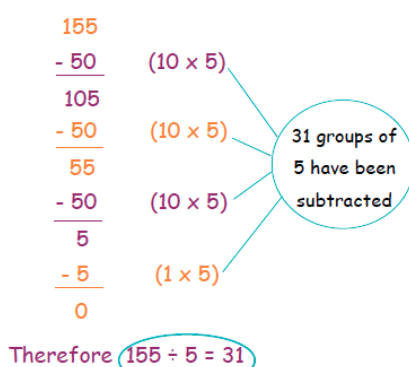
This time you use known multiplication facts to make larger jumps so harder calculations can be solved.

Instead of doing jumps of just 4 (divisor) you can do larger jumps.

This can often be made easier by writing out your known times table facts for the first 10 multiples e.g.  $1 \times 4 = 4$ ,  $2 \times 4 = 8$  etc.

If you cannot count the whole way to the dividend then your answer has a remainder.

### 4. Division through chunking



$$\begin{array}{r} 36r4 \\ 7 \overline{) 256} \\ \underline{- 70} \quad (10 \times 7) \\ \end{array}$$

Answer:  
36 r4

$$\begin{array}{r} 186 \\ - 140 \quad (20 \times 7) \\ \underline{\phantom{00}} 46 \\ - 42 \quad (6 \times 7) \\ \underline{\phantom{00}} 4 \end{array}$$

$$\begin{array}{r} 32r4 \\ 6 \overline{) 196} \\ \underline{- 180} \quad (30 \times 6) \\ \phantom{00} 16 \\ - 12 \quad (2 \times 6) \\ \underline{\phantom{00}} 4 \end{array}$$

Answer:  
32 r4

Division by chunking- this is similar to on a number line but is done in a more formal column method and though subtracting each 'chunked' calculation from the total and repeating.

This can often be made easier by writing out your known times table facts for the first 10 multiples  
E.g.  $1 \times 4 = 4$ ,  $2 \times 4 = 8$  etc.

If you cannot count the whole way to the dividend then your answer has a remainder.

## 5. Drop down method

Step 1: "How many times?"  
Step 2: "Multiply"  
Step 3: "Subtract"  
Step 4: "Drop it down"  
(repeat steps for each number, left to right)

First see how many times your divisor goes into the first digit.

How many 7s go into 4?

The answer is 0.

This then forms a subtraction and you are still left with 4.

Next drop down the following digit.

Now, how many 7s go into 45?

6 whole 7s go which 42, and the 6 goes is above and 42 goes below the available 45 for a subtraction.

Repeat the subtraction and drop down process.

## 6. Short division (Bus stop method)

First see how many times your divisor goes into the first digit.

How many 6s go into 1? The zero gets placed on top.

Now you have to move over anything remaining to the next digit.

The one carries over to the 8.

How many 6s go into 18? The three gets placed on top.

Repeat the process of moving over any remainders for each digit.

If you have anything left over- we call it a remainder.

Remainders can be written as a number, as a fraction or as a decimal.

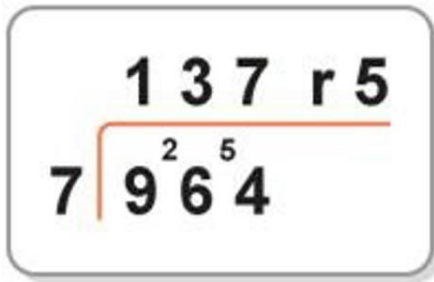
As a fraction it is the left over amount (remainder) over the divisor (denominator) and then simplified to the smallest fraction.

As a decimal you need to put in tenths and hundredths to continue the process as shown below. There were 2 left over from the 142 whole number. Carry this 2 and place with the 0 tenths. How many 4s go into 20?

no groups of 6 can be made  
 $3 \times 6 = 18$   
 $1 \times 6 = 6$

Click [here](#) to see how to turn remainders into decimals and fractions.

## SUMMARY



first.

- Children need to know number and multiplication facts by heart and be tested regularly.

- Children should always estimate

$$142 \div 4 = 35.5$$

$$\begin{array}{r} 035.5 \\ 4 \overline{)142.0} \end{array} \quad \begin{array}{l} r2 \\ 2/4 = 1/2 = 0.5 \end{array}$$

- Thought should be given as to whether a mental method would be more appropriate.
- Attention should be paid to language - referring to the actual value of the digits.
- Answers should always be checked, preferably using a different method, e.g. the inverse operation.
- Errors need to be discussed; problems should be diagnosed and then worked through - do not simply re-teach the method.
- Children who make persistent mistakes should return to the method that they can use accurately until they are ready to move on.
- When extending to harder numbers, refer back to expanded methods. This helps reinforce understanding and reminds children that they have an alternative to fall back on if they are having difficulties.